Type III Compensator Design

For more information on appropriate analog Type III compensator design rules and best practices, please refer to class

23094 PC2: Fundamentals of Switch-Mode Power Supply Control

MICROCHIP
MASTERs 2019

Digital Compensator Design Path

First we select a well fitting, known prototype-filter transfer function (here type III lead-lag compensator)

$$H_c(s) = \frac{\omega_{P0}}{s} \frac{\left(\frac{s}{\omega_{Z1}} + 1\right)\left(\frac{s}{\omega_{Z2}} + 1\right)}{\left(\frac{s}{\omega_{P1}} + 1\right)\left(\frac{s}{\omega_{P2}} + 1\right)}$$

z-Transform

$$s \Rightarrow \frac{2}{T_s} \frac{(1 - z^{-1})}{(1 + z^{-1})}$$

Bi-linear-Transform

Then the s-Domain transfer function of this prototype filter is mapped into the z-domain

$$\boldsymbol{H_C}[z] = \frac{y[z]}{x[z]} = \frac{B_3\ z^{-3} + B_2\ z^{-2} + B_1\ z^{-1} + B_0}{-A_3\ z^{-3} - A_2\ z^{-2} - A_1\ z^{-1} + 1}$$

The BLT gives us equations for the filter coefficients

Coefficient Equations

Ordering z along the delay line then gives us the linear difference equation in time domain

$$u_n = A_1 u_{n-1} + A_2 u_{n-2} + A_3 u_{n-3} + B_0 e_n + B_1 e_{n-1} + B_2 e_{n-2} + B_3 e_{n-3}$$

Digital Filter Design

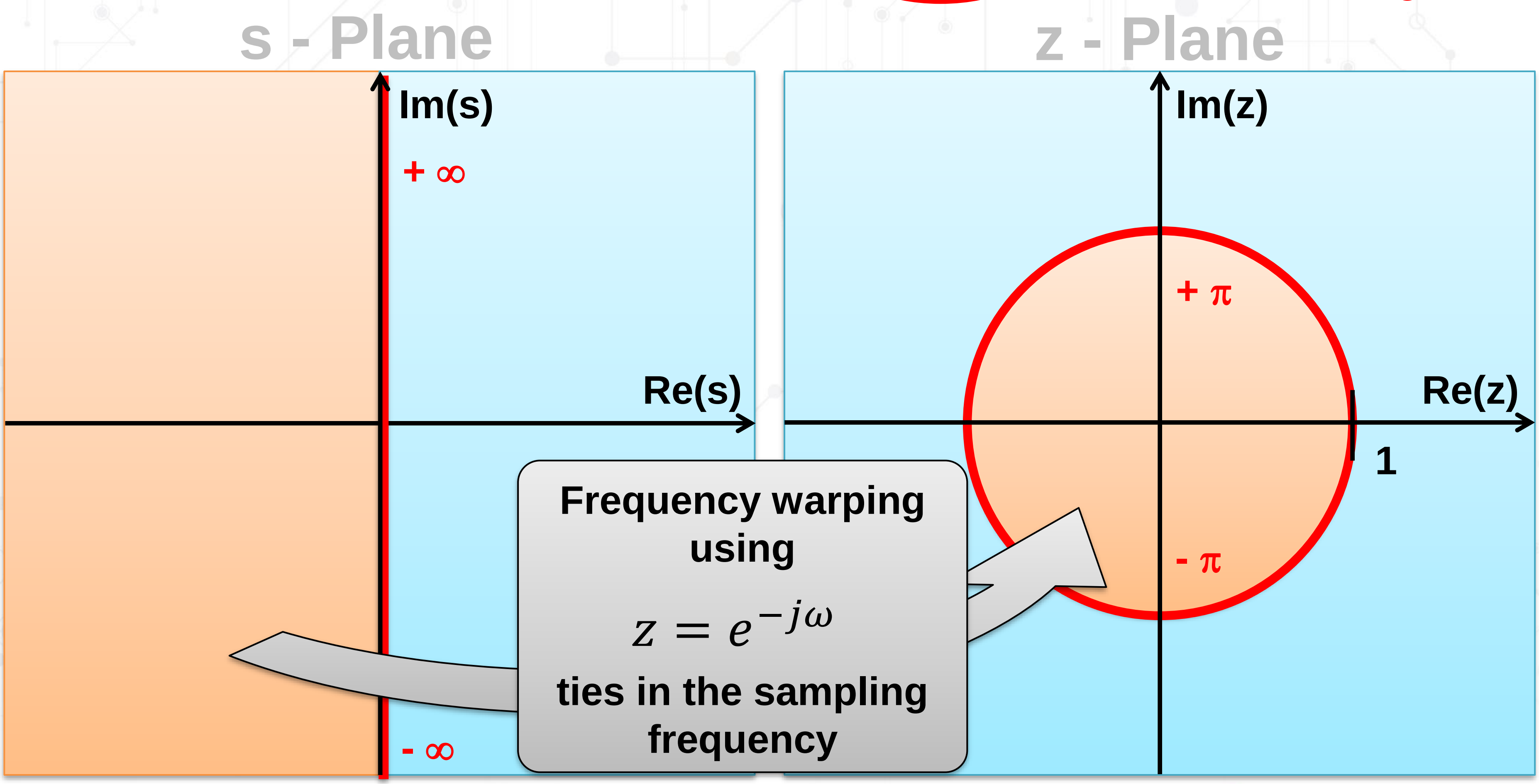
- The Bilinear Transform (BLT) -

Laplace-Transform of the prototype filter

Desired Filter

$$\left|H_c(s) = H(z)\right|_{s=\alpha\frac{1-z^{-1}}{1+z^{-1}}}$$

Bilinear Transform (BLT) where $\alpha = 2/T_S$



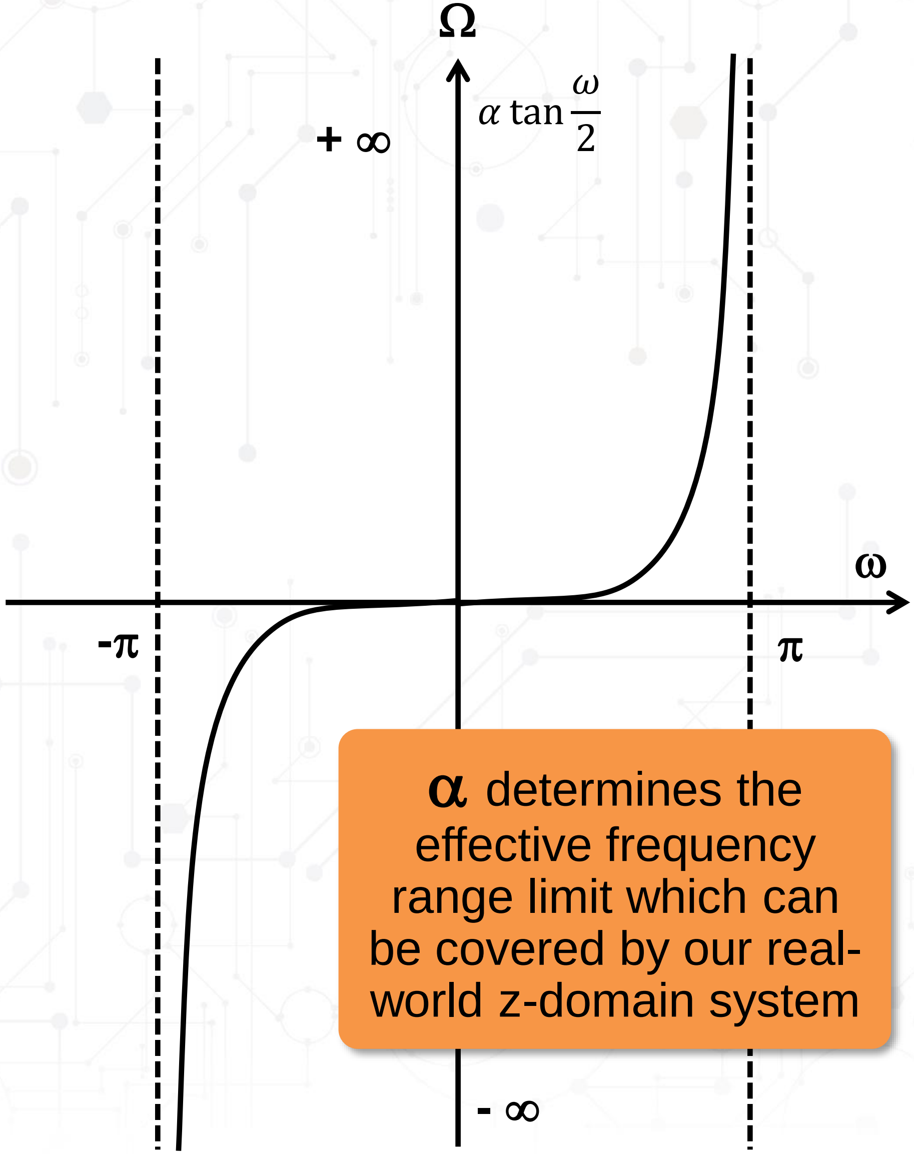
Digital Filter Design

By considering the sampling frequency of the digital filter, we now can prove, that the digital frequency response of the desired filter matches the frequency response of the analog prototype filter

$$H_D(\omega) = H(e^{-j\omega}) = H_c(s)\Big|_{s=\alpha\frac{1-z^{-1}}{1+z^{-1}}}$$

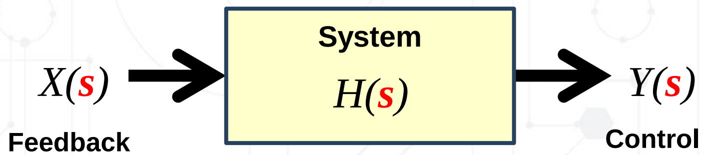
$$H_D(\omega) = H_A\left(\alpha \tan\frac{\omega}{2}\right)$$

This is important as this is the prove that we can map an *infinite* frequency space from -∞ to ∞ onto a digital frequency space bounded between –π and π



α determines the effective frequency range limit which can be covered by our real-world z-domain system

Bilinear Transform



$$H(s) = \frac{Y(s)}{X(s)}$$

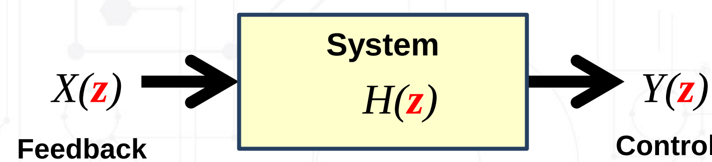
$$Y(s) = H(s) \times X(s)$$

All we need to do is to replace s operators in H(s) with:

$$s \Rightarrow \frac{2}{T_s} \frac{(1 - z^{-1})}{(1 + z^{-1})}$$

Where T_S = sampling interval = $1/f_S$

- Tustin or Trapezoidal
- It converts an analog transfer function in *s* domain into an equivalent digital transfer function in *z* domain
- It is an approximation (!!!)
- The lower the cross over frequency with respect to your sampling frequency the better the approximation
- For conservative design $f_X \leq f_S / 10$



$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = H(z) \times X(z)$$

Designing a 3P3Z Controller

Type III lead-lag compensator

$$H_c(s) = \frac{\omega_{P0}}{s}\frac{\left(\frac{s}{\omega_{Z1}}+1\right)\left(\frac{s}{\omega_{Z2}}+1\right)}{\left(\frac{s}{\omega_{P1}}+1\right)\left(\frac{s}{\omega_{P2}}+1\right)}$$

with $s \Rightarrow \frac{2}{T_s}\frac{(1 - z^{-1})}{(1 + z^{-1})}$

We get:

$$H\left(\frac{2\left(1-\frac{1}{z}\right)}{T\left(\frac{1}{z}+1\right)}\right) = \frac{T\boldsymbol{\omega}_{\mathbf{P0}}\left(\frac{2\left(1-\frac{1}{z}\right)}{T\boldsymbol{\omega}_{\mathbf{Z1}}\left(\frac{1}{z}+1\right)}+1\right)\left(\frac{2\left(1-\frac{1}{z}\right)}{T\boldsymbol{\omega}_{\mathbf{Z2}}\left(\frac{1}{z}+1\right)}+1\right)\left(\frac{1}{z}+1\right)}{2\left(\frac{2\left(1-\frac{1}{z}\right)}{T\boldsymbol{\omega}_{\mathbf{P1}}\left(\frac{1}{z}+1\right)}+1\right)\left(\frac{2\left(1-\frac{1}{z}\right)}{T\boldsymbol{\omega}_{\mathbf{P2}}\left(\frac{1}{z}+1\right)}+1\right)\left(1-\frac{1}{z}\right)}$$

This term now needs to be factorized to get us to the desired polynomial form

Designing a 3P3Z Controller

… and this is when things start to get a bit messy for a while …

$$
\begin{aligned}
&\frac{2T\mathrm{wp0wp1wp2}z^3}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1wz2}}\\
&-\frac{2T\mathrm{wp0wp1wp2}z^2}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1wz2}}\\
&-\frac{2T\mathrm{wp0wp1wp2}z}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1wz2}}\\
&+\frac{2T\mathrm{wp0wp1wp2}}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1wz2}}\\
&+\frac{T^2\mathrm{wp0wp1wp2}z^3}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz2}}\\
&+\frac{T^2\mathrm{wp0wp1wp2}z^2}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz2}}\\
&-\frac{T^2\mathrm{wp0wp1wp2}z}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz2}}\\
&-\frac{T^2\mathrm{wp0wp1wp2}}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz2}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz2}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz2}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz2}}\\
&+\frac{T^2\mathrm{wp0wp1wp2}z^3}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1}}\\
&+\frac{T^2\mathrm{wp0wp1wp2}z^2}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1}}\\
&-\frac{T^2\mathrm{wp0wp1wp2}z}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1}}\\
&-\frac{T^2\mathrm{wp0wp1wp2}}{((T^2\mathrm{wp1}+2T)\mathrm{wp2}+2T\mathrm{wp1}+4)\mathrm{wz1}z^3+((T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}-12)\mathrm{wz1}z^2+((-T^2\mathrm{wp1}-2T)\mathrm{wp2}-2T\mathrm{wp1}+12)\mathrm{wz1}z+((2T-T^2\mathrm{wp1})\mathrm{wp2}+2T\mathrm{wp1}-4)\mathrm{wz1}}\\
&+\frac{T^3\mathrm{wp0wp1wp2}z^3}{2T^2\mathrm{wp1wp2}z^3+4T\mathrm{wp2}z^3+4T\mathrm{wp1}z^3+8z^3+2T^2\mathrm{wp1wp2}z^2-4T\mathrm{wp2}z^2-4T\mathrm{wp1}z^2-24z^2-2T^2\mathrm{wp1wp2}z-4T\mathrm{wp2}z-4T\mathrm{wp1}z+24z-2T^2\mathrm{wp1wp2}+4T\mathrm{wp2}+4T\mathrm{wp1}-8}\\
&+\frac{3T^3\mathrm{wp0wp1wp2}z^2}{2T^2\mathrm{wp1wp2}z^3+4T\mathrm{wp2}z^3+4T\mathrm{wp1}z^3+8z^3+2T^2\mathrm{wp1wp2}z^2-4T\mathrm{wp2}z^2-4T\mathrm{wp1}z^2-24z^2-2T^2\mathrm{wp1wp2}z-4T\mathrm{wp2}z-4T\mathrm{wp1}z+24z-2T^2\mathrm{wp1wp2}+4T\mathrm{wp2}+4T\mathrm{wp1}-8}\\
&+\frac{3T^3\mathrm{wp0wp1wp2}z}{2T^2\mathrm{wp1wp2}z^3+4T\mathrm{wp2}z^3+4T\mathrm{wp1}z^3+8z^3+2T^2\mathrm{wp1wp2}z^2-4T\mathrm{wp2}z^2-4T\mathrm{wp1}z^2-24z^2-2T^2\mathrm{wp1wp2}z-4T\mathrm{wp2}z-4T\mathrm{wp1}z+24z-2T^2\mathrm{wp1wp2}+4T\mathrm{wp2}+4T\mathrm{wp1}-8}\\
&+\frac{T^3\mathrm{wp0wp1wp2}}{2T^2\mathrm{wp1wp2}z^3+4T\mathrm{wp2}z^3+4T\mathrm{wp1}z^3+8z^3+2T^2\mathrm{wp1wp2}z^2-4T\mathrm{wp2}z^2-4T\mathrm{wp1}z^2-24z^2-2T^2\mathrm{wp1wp2}z-4T\mathrm{wp2}z-4T\mathrm{wp1}z+24z-2T^2\mathrm{wp1wp2}+4T\mathrm{wp2}+4T\mathrm{wp1}-8}
\end{aligned}
$$

Luckily *symbolic* equation solvers like Mathematica/WolframAlpha Online, Maple, Reduce, Maxima, etc… can help (!!!)

Designing a 3P3Z Controller

After factorizing the term, we now start to see the finish line…

$$H_C[z] = \frac{\begin{array}{r}\omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1}+2T^2)\omega_{Z2}+2T^2\omega_{Z1}+4T\right)z^3+\\ \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1}+2T^2)\omega_{Z2}+2T^2\omega_{Z1}-4T\right)z^2+\\ \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1}-2T^2)\omega_{Z2}-2T^2\omega_{Z1}-4T\right)z^1+\\ \omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1}-2T^2)\omega_{Z2}-2T^2\omega_{Z1}+4T\right)z^0\end{array}}{\begin{array}{r}\omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}+4T)\omega_{P2}+4T\omega_{P1}+8\right)z^3+\\ \omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}-4T)\omega_{P2}-4T\omega_{P1}-24\right)z^2+\\ \omega_{Z1}\omega_{Z2}\left((-2T^2\omega_{P1}-4T)\omega_{P2}-4T\omega_{P1}+24\right)z^1+\\ \omega_{Z1}\omega_{Z2}\left((4T-2T^2\omega_{P1})\omega_{P2}+4T\omega_{P1}-8\right)z^0\end{array}}$$

$$H_C[z] = \frac{B_3\,z^{-3} + B_2\,z^{-2} + B_1\,z^{-1} + B_0}{-A_3\,z^{-3} - A_2\,z^{-2} - A_1\,z^{-1} + 1}$$

Designing a 3P3Z Controller

It is ***almost*** in the right form, leaving us with ***two remaining problems*** to solve…

$$H_C[z] = \frac{\begin{array}{r} \omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1} + 2T^2)\omega_{Z2} + 2T^2\omega_{Z1} + 4T\right) z^3 + \\ \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1} + 2T^2)\omega_{Z2} + 2T^2\omega_{Z1} - 4T\right) z^2 + \\ \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1} - 2T^2)\omega_{Z2} - 2T^2\omega_{Z1} - 4T\right) z^1 + \\ \omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1} - 2T^2)\omega_{Z2} - 2T^2\omega_{Z1} + 4T\right) z^0 \end{array}}{\begin{array}{r} \omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1} + 4T)\omega_{P2} + 4T\omega_{P1} + 8\right) z^3 + \\ \omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1} - 4T)\omega_{P2} - 4T\omega_{P1} - 24\right) z^2 + \\ \omega_{Z1}\omega_{Z2}\left((-2T^2\omega_{P1} - 4T)\omega_{P2} - 4T\omega_{P1} + 24\right) z^1 + \\ \omega_{Z1}\omega_{Z2}\left((4T - 2T^2\omega_{P1})\omega_{P2} + 4T\omega_{P1} - 8\right) z^0 \end{array}}$$

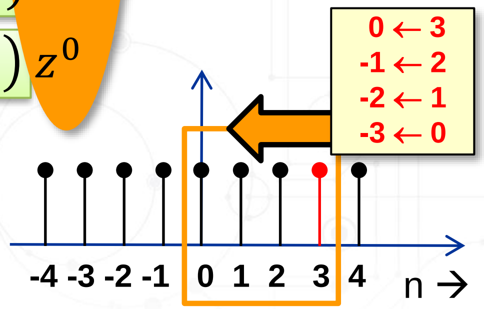
$$H_C[z] = \frac{B_3\, z^{-3} + B_2\, z^{-2} + B_1\, z^{-1} + B_0}{-A_3\, z^{-3} - A_2\, z^{-2} - A_1\, z^{-1} + 1}$$

Problem #1:

Relocating the delay line.

This term is pointing "into the future".

Therefore the entire term needs to be shifted three ticks "into the past"



Designing a 3P3Z Controller

Step 1:

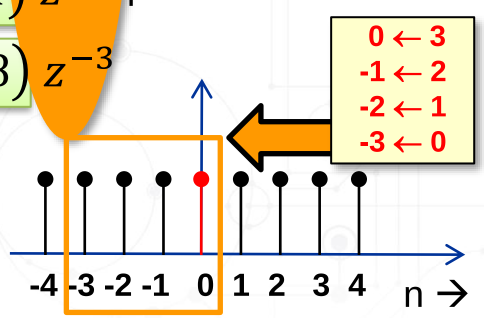
Moving delay line by 3-clicks "into the past" synchronizes equation and our target transfer function form

$\cancel{B_0 =}$ $\cancel{B_1 =}$ $\cancel{B_2 =}$ $\cancel{B_3 =}$ $\cancel{A_0 =}$ $\cancel{A_1 =}$ $\cancel{A_2 =}$ $\cancel{A_3 =}$

NOT YET !

$$= \frac{\begin{gathered}\omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1}+2T^2)\omega_{Z2}+2T^2\omega_{Z1}+4T\right)z^{0}+\\ \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1}+2T^2)\omega_{Z2}+2T^2\omega_{Z1}-4T\right)z^{-1}+\\ \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1}-2T^2)\omega_{Z2}-2T^2\omega_{Z1}-4T\right)z^{-2}+\\ \omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1}-2T^2)\omega_{Z2}-2T^2\omega_{Z1}+4T\right)z^{-3}\end{gathered}}{\begin{gathered}\omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}+4T)\omega_{P2}+4T\omega_{P1}+8\right)z^{0}+\\ \omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}-4T)\omega_{P2}-4T\omega_{P1}-24\right)z^{-1}+\\ \omega_{Z1}\omega_{Z2}\left((-2T^2\omega_{P1}-4T)\omega_{P2}-4T\omega_{P1}+24\right)z^{-2}+\\ \omega_{Z1}\omega_{Z2}\left((4T-2T^2\omega_{P1})\omega_{P2}+4T\omega_{P1}-8\right)z^{-3}\end{gathered}}$$

$$H_C[z] = \frac{B_3\, z^{-3} + B_2\, z^{-2} + B_1\, z^{-1} + B_0}{-A_3\, z^{-3} - A_2\, z^{-2} - A_1\, z^{-1} + 1}$$



Designing a 3P3Z Controller

Step 2:

The entire term needs to be ***normalized*** to make coefficient A_0 = 1

$$\begin{aligned}
B_0 &= \omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1}+2T^2)\omega_{Z2}+2T^2\omega_{Z1}+4T\right)z^0+\\
B_1 &= \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1}+2T^2)\omega_{Z2}+2T^2\omega_{Z1}-4T\right)z^{-1}+\\
B_2 &= \omega_{P0}\omega_{P1}\omega_{P2}\left((3T^3\omega_{Z1}-2T^2)\omega_{Z2}-2T^2\omega_{Z1}-4T\right)z^{-2}+\\
B_3 &= \omega_{P0}\omega_{P1}\omega_{P2}\left((T^3\omega_{Z1}-2T^2)\omega_{Z2}-2T^2\omega_{Z1}+4T\right)z^{-3}
\end{aligned}$$

$$\begin{aligned}
A_0 &= \omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}+4T)\omega_{P2}+4T\omega_{P1}+8\right)z^0+\\
A_1 &= \omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}-4T)\omega_{P2}-4T\omega_{P1}-24\right)z^{-1}+\\
A_2 &= \omega_{Z1}\omega_{Z2}\left((-2T^2\omega_{P1}-4T)\omega_{P2}-4T\omega_{P1}+24\right)z^{-2}+\\
A_3 &= \omega_{Z1}\omega_{Z2}\left((4T-2T^2\omega_{P1})\omega_{P2}+4T\omega_{P1}-8\right)z^{-3}
\end{aligned}$$

NOT YET !

$$H_C[z] = \frac{B_3\,z^{-3} + B_2\,z^{-2} + B_1\,z^{-1} + B_0}{-A_3\,z^{-3} - A_2\,z^{-2} - A_1\,z^{-1} + 1}$$

Problem #2:

Coefficient A0 needs to become const. 1, which it is clearly not

Designing a 3P3Z Controller

Step 2:

The entire term needs to be ***normalized*** to make coefficient A_0 **= 1**

So we perform the following multiplication

$$H_C[z] \times \frac{\dfrac{1}{\omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}+4T)\omega_{P2}+4T\omega_{P1}+8\right)}}{-\dfrac{1}{\omega_{Z1}\omega_{Z2}\left((2T^2\omega_{P1}+4T)\omega_{P2}+4T\omega_{P1}+8\right)}}$$

and we get…

Designing a 3P3Z Controller

CHECKLIST

- ☑ Delay-Line correct?
- ☑ $A_0 = 1$?
- ☑ Sign of A-Coefficients correct?

That's it !!!

$$B_0 = \frac{T\omega_{P0}\omega_{P1}\omega_{P2}(T\omega_{Z1}+2)(T\omega_{Z2}+2)}{2(T\omega_{P1}+2)(T\omega_{P2}+2)\omega_{Z1}\omega_{Z2}} z^0 +$$

$$B_1 = \frac{T\omega_{P0}\omega_{P1}\omega_{P2}\left(-4+3T^2\omega_{Z1}\omega_{Z2}+2T(\omega_{Z1}+\omega_{Z2})\right)}{2(T\omega_{P1}+2)(T\omega_{P2}+2)\omega_{Z1}\omega_{Z2}} z^{-1} +$$

$$B_2 = \frac{T\omega_{P0}\omega_{P1}\omega_{P2}\left(-4+3T^2\omega_{Z1}\omega_{Z2}-2T(\omega_{Z1}+\omega_{Z2})\right)}{2\omega_{Z1}\omega_{Z2}(T\omega_{P1}+2)(T\omega_{P2}+2)} z^{-2} +$$

$$B_3 = \frac{\left(T\omega_{P0}\omega_{P1}\omega_{P2}(T\omega_{Z1}-2)(T\omega_{Z2}-2)\right)}{2(T\omega_{P1}+2)(T\omega_{P2}+2)\omega_{Z1}\omega_{Z2}} z^{-3}$$

$$A_0 = 1 \; z^0 +$$

$$A_1 = -\frac{-12+T^2\omega_{P1}\omega_{P2}-2T(\omega_{P1}+\omega_{P2})}{(T\omega_{P1}+2)(T\omega_{P2}+2)} z^{-1} +$$

$$A_2 = \frac{-12+T^2\omega_{P1}\omega_{P2}+2T(\omega_{P1}+\omega_{P2})}{(T\omega_{P1}+2)(T\omega_{P2}+2)} z^{-2} +$$

$$A_3 = \frac{(T\omega_{P1}-2)(T\omega_{P2}-2)}{(T\omega_{P1}+2)(T\omega_{P2}+2)} z^{-3}$$

Designing a 3P3Z Controller

Now we've got a 100% generic compensator equation which can be set up by applying common s-domain design rules and techniques!

$$H[z] = \frac{y[z]}{x[z]} = \frac{B_3\ z^{-3} + B_2\ z^{-2} + B_1\ z^{-1} + B_0}{-A_3\ z^{-3} - A_2\ z^{-2} - A_1\ z^{-1} + 1}$$

with

$$A_1 = -\frac{\left(-12 + T_S^2\omega_{P1}\omega_{P2} - 2T_S(\omega_{P1} + \omega_{P2})\right)}{(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})}$$

$$A_2 = \frac{\left(-12 + T_S^2\omega_{P1}\omega_{P2} + 2T_S(\omega_{P1} + \omega_{P2})\right)}{(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})}$$

$$A_3 = \frac{(-2 + T_S\omega_{P1})(-2 + T_S\omega_{P2})}{(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})}$$

$$B_0 = \frac{\left(T_S\omega_{P0}\omega_{P1}\omega_{P2}(2 + T_S\omega_{Z1})(2 + T_S\omega_{Z2})\right)}{\left(2\omega_{Z1}\omega_{Z2}(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})\right)}$$

$$B_1 = \frac{\left(T_S\omega_{P0}\omega_{P1}\omega_{P2}\left(-4 + 3T_S^2\omega_{Z1}\omega_{Z2} + 2T_S(\omega_{Z1} + \omega_{Z2})\right)\right)}{\left(2\omega_{Z1}\omega_{Z2}(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})\right)}$$

$$B_2 = \frac{\left(T_S\omega_{P0}\omega_{P1}\omega_{P2}\left(-4 + 3T_S^2\omega_{Z1}\omega_{Z2} - 2T_S(\omega_{Z1} + \omega_{Z2})\right)\right)}{\left(2\omega_{Z1}\omega_{Z2}(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})\right)}$$

$$B_3 = \frac{\left(T_S\omega_{P0}\omega_{P1}\omega_{P2}(-2 + T_S\omega_{Z1})(-2 + T_S\omega_{Z2})\right)}{\left(2\omega_{Z1}\omega_{Z2}(2 + T_S\omega_{P1})(2 + T_S\omega_{P2})\right)}$$

Enrolling the z-Transfer Function on the Delay Line

$$\frac{y[z]}{x[z]} = \frac{B_3 z^{-3} + B_2 z^{-2} + B_1 z^{-1} + B_0}{-A_3 z^{-3} - A_2 z^{-2} - A_1 z^{-1} + 1}$$

$$x[z] \times (B_3 z^{-3} + B_2 z^{-2} + B_1 z^{-1} + B_0) = y[z] \times (-A_3 z^{-3} - A_2 z^{-2} - A_1 z^{-1} + 1)$$

$$(B_3 x_{n-3} + B_2 x_{n-2} + B_1 x_{n-1} + B_0 x_n) = (-A_3 y_{n-3} - A_2 y_{n-2} - A_1 y_{n-1} + 1 y_n)$$

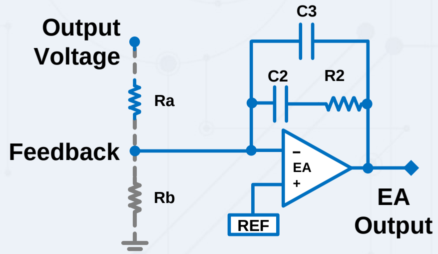
Here is our next control output!

$$\boxed{y_n = +A_3 y_{n-3} + A_2 y_{n-2} + A_1 y_{n-1} + B_3 x_{n-3} + B_2 x_{n-2} + B_1 x_{n-1} + B_0 x_n}$$

This LDE can now run on the DSP core most efficiently

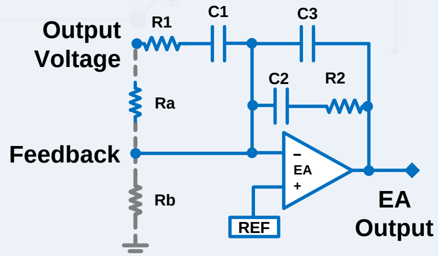
New Degree of Control Flexibility

Type II



$$H_c(s) = \frac{\omega_{P0}}{s} \frac{\left(\frac{s}{\omega_{Z1}} + 1\right)}{\left(\frac{s}{\omega_{P1}} + 1\right)}$$

Type III



$$H_c(s) = \frac{\omega_{P0}}{s} \frac{\left(\frac{s}{\omega_{Z1}} + 1\right)\left(\frac{s}{\omega_{Z2}} + 1\right)}{\left(\frac{s}{\omega_{P1}} + 1\right)\left(\frac{s}{\omega_{P2}} + 1\right)}$$

Type IV

?

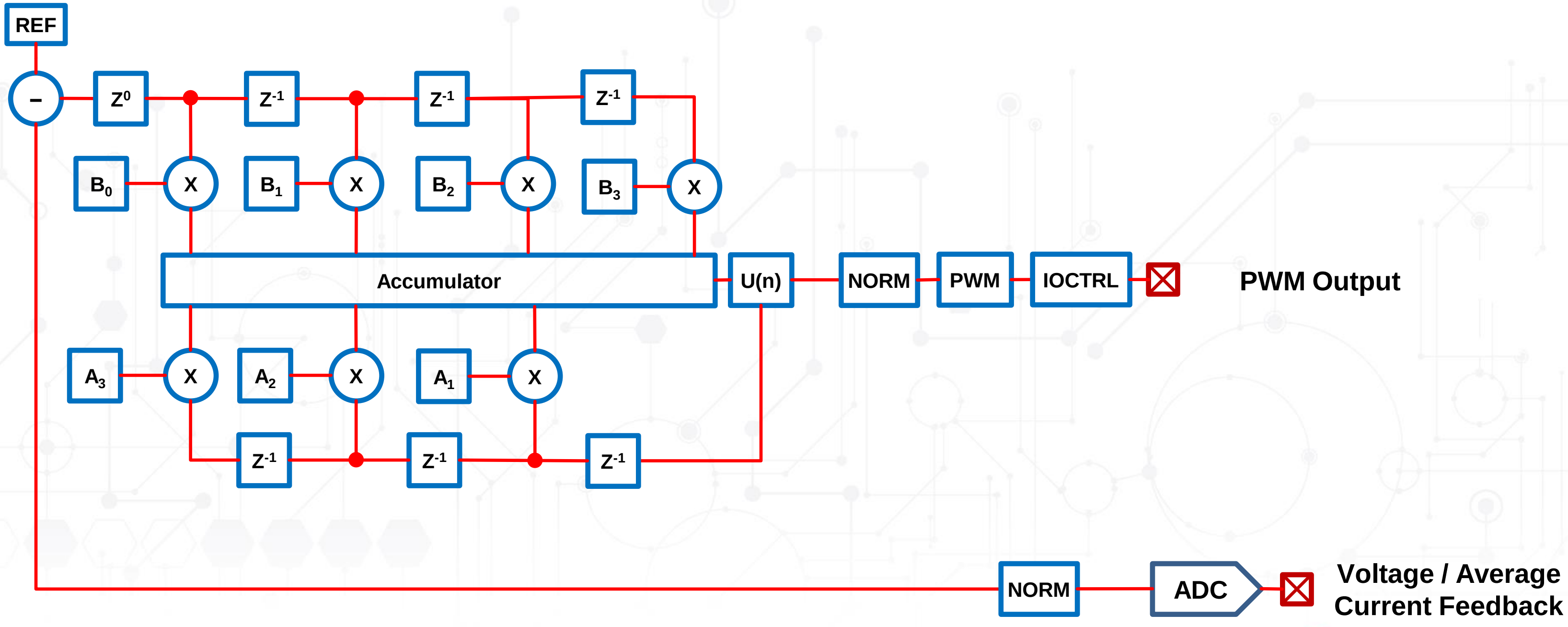
$$H_c(s) = \frac{\omega_{P0}}{s} \frac{\left(\frac{s}{\omega_{Z1}} + 1\right)\left(\frac{s}{\omega_{Z2}} + 1\right)\left(\frac{s}{\omega_{Z3}} + 1\right)}{\left(\frac{s}{\omega_{P1}} + 1\right)\left(\frac{s}{\omega_{P2}} + 1\right)\left(\frac{s}{\omega_{P3}} + 1\right)}$$

Type XII

$$H_c(s) = \frac{\omega_{P0}}{s} \frac{\left(\frac{s}{\omega_{Z1}} + 1\right)\left(\frac{s}{\omega_{Z2}} + 1\right)\left(\frac{s}{\omega_{Z3}} + 1\right)\left(\frac{s}{\omega_{Z4}} + 1\right)\left(\frac{s}{\omega_{Z5}} + 1\right)\left(\frac{s}{\omega_{Z6}} + 1\right)\left(\frac{s}{\omega_{Z7}} + 1\right)\left(\frac{s}{\omega_{Z8}} + 1\right)\left(\frac{s}{\omega_{Z9}} + 1\right)\left(\frac{s}{\omega_{Z10}} + 1\right)\left(\frac{s}{\omega_{Z11}} + 1\right)}{\left(\frac{s}{\omega_{P1}} + 1\right)\left(\frac{s}{\omega_{P2}} + 1\right)\left(\frac{s}{\omega_{P3}} + 1\right)\left(\frac{s}{\omega_{P4}} + 1\right)\left(\frac{s}{\omega_{P5}} + 1\right)\left(\frac{s}{\omega_{P6}} + 1\right)\left(\frac{s}{\omega_{P7}} + 1\right)\left(\frac{s}{\omega_{P8}} + 1\right)\left(\frac{s}{\omega_{P9}} + 1\right)\left(\frac{s}{\omega_{P10}} + 1\right)\left(\frac{s}{\omega_{P11}} + 1\right)}$$

Digital Type III Controller
– Basic Implementation –

- This is the block diagram of the common, fully generic 3p3z compensator we just designed
- Its output is normalized to provide a PWM duty cycle, phase-shift, switching period, reference current, amplitude modulation factor, etc.



Digital Control Loop Implementation