Loop Gain Measurement

The Voltage Injection Method using the Bode 100 and the B-WIT 100

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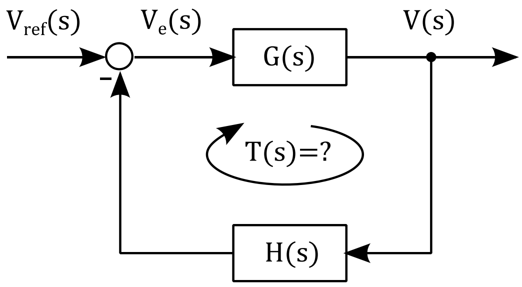
1 Introduction

Negative feedback is commonly used for control systems.
The figure below shows a simple system with a negative feedback.



The output signal or voltage $V(s)$ follows the reference voltage $V_{ref}(s)$ by the relation

$$\frac{V(s)}{V_{ref}(s)} = \frac{G(s)}{1 + T(s)}$$

This is denoted as the "closed loop transfer function".
$T(s)$ is called the "loop gain" which is the product of all gains around the loop and equals in this case to $T(s) = H(s)G(s)$.

Knowing the loop gain one can apply the Nyquist stability criterion to measure the gain and phase margin and assess the overall stability of the closed loop system.

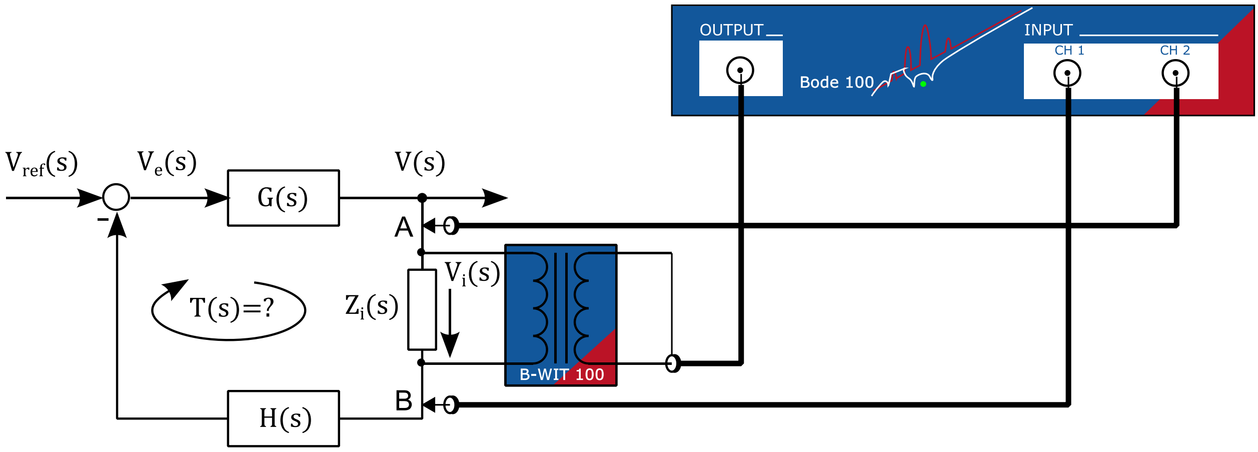
The loop gain of a system can e.g. be derived from a model of the system. Models normally do not consider all parasitics and unwanted effects. Therefore it can be advantageous to measure the loop gain of a feedback system during the design process.

2 Loop Gain Measurement

One practical method to measure the loop gain in electronic feedback systems like DC-DC converters or voltage regulators is the voltage injection method (1). The theoretical derivation of the voltage injection method is shown in the next section. In the following is shown how the voltage injection method can be applied in practice and what has to be considered to achieve correct results.

Using a suitable injection transformer (e.g. the B-WIT 100) one can inject a test voltage at an appropriate injection-point in the feedback loop of the system. Then the response of the loop can be measured using a vector network analyzer or a frequency response analyzer like the Bode 100.

The following figure shows the principle measurement setup to measure the loop gain of a feedback system. A resistor $Z_i(s) \approx 10\Omega$ is inserted in the feedback loop at an appropriate injection-point. The injection transformer is connected in parallel to the injection resistor to apply the test voltage $V_i(s)$ at the injection resistor. This enables the injection of the test voltage without changing the DC-bias of the system.



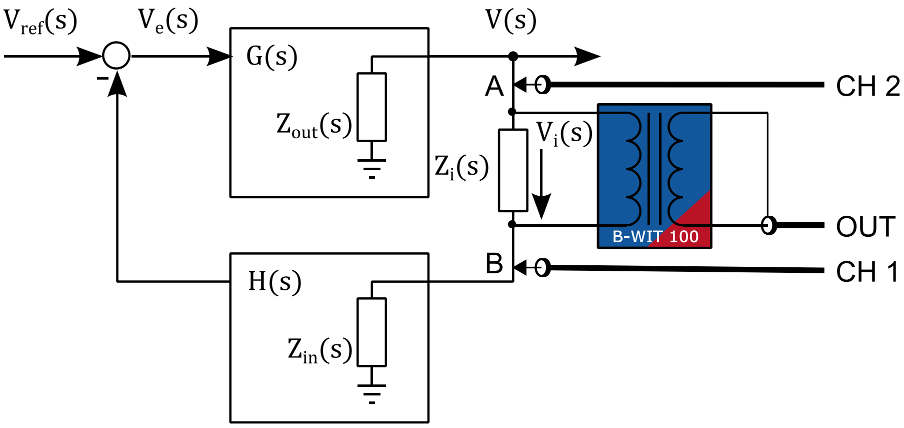
The analyzers inputs are connected on both sides of the injection resistor using coaxial cables or voltage probes. The loop gain is then measured by measuring the complex voltage gain from point A to B.

$$T_v(s) = \frac{V_{CH2}(s)}{V_{CH1}(s)}$$

Where T_v is the measured loop gain and V_{CH1} and V_{CH2} are measured voltages.

The measured loop gain $T_v(s)$ equals approximately the "real" loop gain if the impedance looking forward around the feedback loop $Z_{in}(s)$ is much greater than the impedance looking backwards from the injection point $Z_{out}(s)$.

$$|Z_{in}(s)| \gg |Z_{out}(s)| \qquad \text{Condition 1}$$



The second condition that must be fulfilled to ensure that the measured loop gain equals approximately the real loop gain is:

$$|T(s)| \gg \left|\frac{Z_{out}(s)}{Z_{in}(s)}\right| \qquad \text{Condition 2}$$

From this we see that it is important to choose a suitable injection point that fulfills both conditions.

The first condition $|Z_{in}| \gg |Z_{out}|$ is often fulfilled at the output of the voltage regulator which is normally of low impedance characteristic. Further suitable points are generally at high impedance inputs like operational amplifier inputs.

The second condition is more difficult to check. Especially small loop gain results, above the crossover frequency need to be checked very carefully.

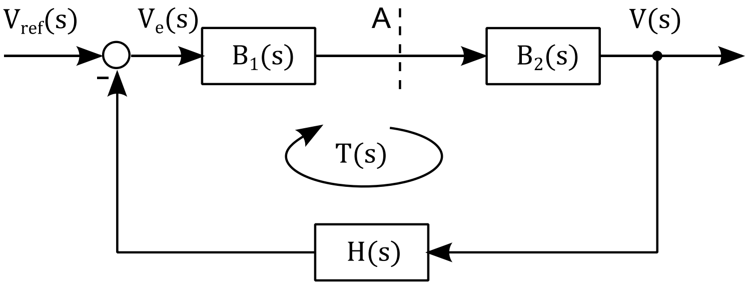
The **magnitude** of the injection voltage should be kept as low as possible to avoid large signal effects as saturation or nonlinearities influence the measurement.

The size of the injection **resistor** does not directly influence the measurement result if it is kept small enough. We recommend using a 10Ω resistor in combination with the B-WIT 100 to ensure the use of the full measurement frequency range from 1Hz to 10MHz.

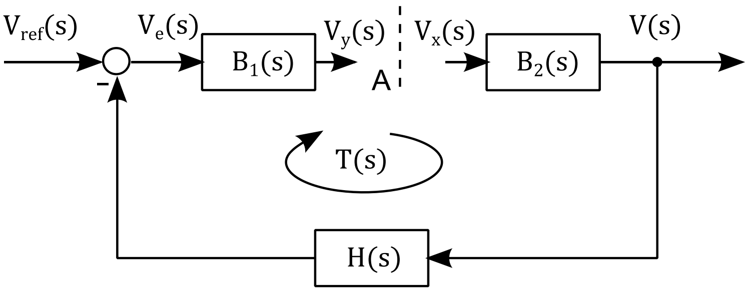
In the following the derivation of the two conditions $|Z_{in}(s)| \gg |Z_{out}(s)|$ and $|T(s)| \gg \left|\frac{Z_{out}(s)}{Z_{in}(s)}\right|$ is shown according to (2).

3 The Voltage Injection Method

In general the loop gain $T(s)$ of a feedback system as shown below can be measured by breaking the feedback loop at a suitable point A where two blocks of the feedback system are connected electrically.

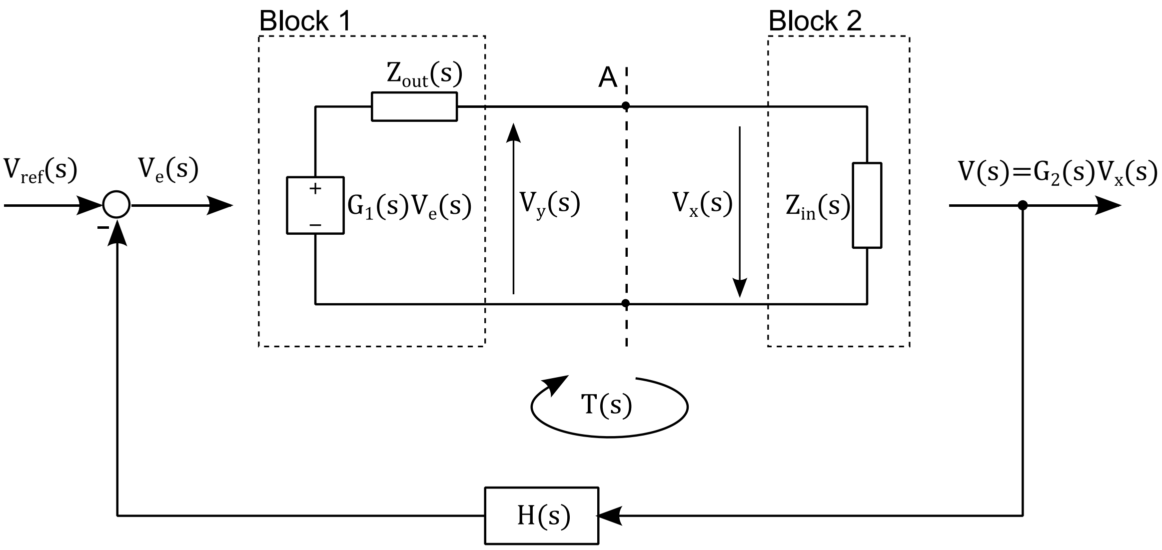


Applying a test voltage $V_x(s)$ after the injection point A leads to the response voltage $V_y(s)$ before the injection point.



The measured loop gain then equals $T_m(s) = \frac{V_y(s)}{V_x(s)}$.

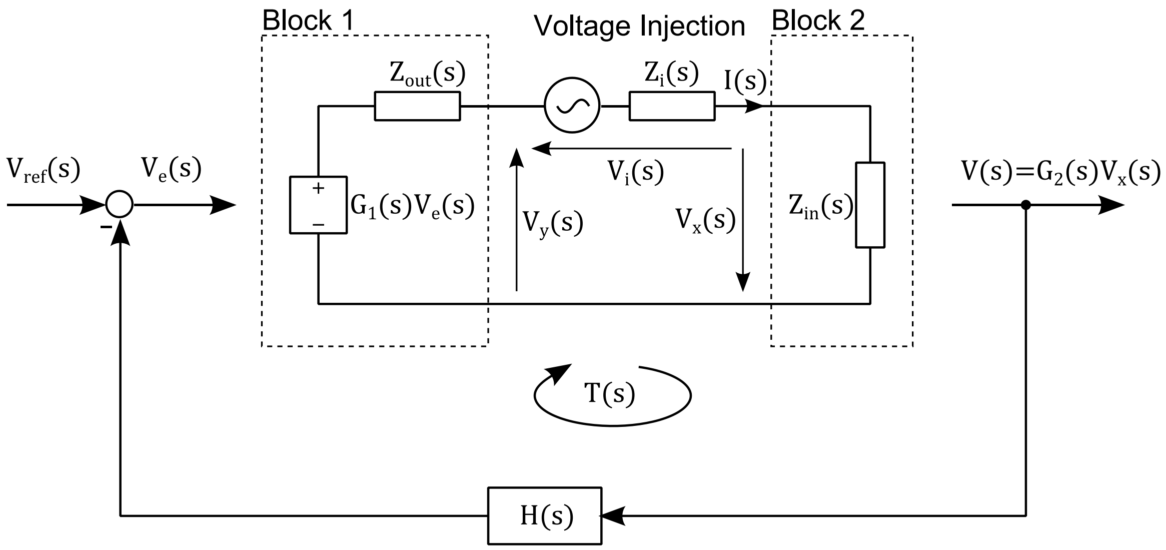
Lets have a closer look at the injection point A where the two blocks B_1 and B_2 are connected electrically and lets extend our model. The output of the first block is modelled with a dependent voltage source $G_1 V_e$ including the source impedance Z_{out}. The input of the second block is modelled with an input impedance Z_{in}:



Using the model from above it can be seen that the input impedance of the second block $Z_{in}(s)$ loads the output of the first block. Considering this model which includes the impedances Z_{in} and Z_{out}, the "real" loop gain $T(s)$ of the system can be calculated by:

$$T(s) = G_1(s)\frac{Z_{in}(s)}{Z_{out}(s) + Z_{in}(s)}G_2(s)H(s) \quad (1)$$

The voltage injection is now included in the model as shown in the following figure



The injecting source is modeled with a voltage source and a series impedance $Z_i(s)$.

In order to measure the loop gain our vector network analyzer is connected to the system and measures

$$T_v(s) = \frac{V_y(s)}{V_x(s)}$$

All other inputs of the system (reference voltage and supply voltage) are considered to be constant showing no AC variations.

According to this model the error voltage is then given by

$$V_e(s) = -H(s)G_2(s)V_x(s)$$

Considering the current flowing from block 1 to block 2, the output voltage of block 1 can be written as

$$-V_y(s) = G_1(s)V_e(s) - I(s)Z_{out}(s)$$

Combining these two equations leads to

$$V_y(s) = V_x(s)G_1(s)G_2(s)H(s) - I(s)Z_{out}(s) \quad (2)$$

The current $I(s)$ can be expressed by

$$I(s) = \frac{V_x(s)}{Z_{in}(s)}$$

Therefore expression (2) becomes

$$V_y(s) = V_x(s)G_1(s)G_2(s)H(s) - V_x(s)\frac{Z_{out}(s)}{Z_{in}(s)}$$

Hence the measured loop gain equals

$$T_v(s) = \frac{V_y(s)}{V_x(s)} = G_1(s)G_2(s)H(s) + \frac{Z_{out}(s)}{Z_{in}(s)}$$

Expressing the measured loop gain $T_v(s)$ in term of the acutal loop gain $T(s)$ from (1) leads to

$$T_v(s) = T(s)\underbrace{\left(1 + \frac{Z_{out}(s)}{Z_{in}(s)}\right)}_{1^{\text{st}}\text{ term}} + \underbrace{\frac{Z_{out}(s)}{Z_{in}(s)}}_{2^{\text{nd}}\text{ term}}$$

The first term of this expression is proportional to the actual loop gain $T(s)$ and is approximately equal to $T(s)$ when

$$|Z_{in}(s)| \gg |Z_{out}(s)|$$

Which is the first condition necessary to fulfill $T_v(s) \approx T(s)$.

The second term limits the minimum loop gain that can be measured using the voltage injection method. If this term is much smaller than the loop gain it can be ignored. Thus we arrive at the second condition to ensure $T_v(s) \approx T(s)$:

$$|T(s)| \gg \left|\frac{Z_{out}(s)}{Z_{in}(s)}\right|$$

The injection resistor respectively injection voltage does influence the loading from block 2 on block 1. However, if condition 1 is satisfied and the test signal is kept small comparing to the absolute signal, this effect is negligible.

References

1. *Measurement of loop gain in feedback systems.* **Middlebrook, R.D.** s.l. : International Journal of Electronics, 1975, Bd. 38.
2. **Erickson, Robert W. und Maksimovic, Dragan.** *Fundamentals of Power Electronics.* s.l. : Springer, 2001.